GUIDELINE FOR EXCEL TOOL TO CALCULATE REINFORCED CONCRETE PLASTER

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Summary

Instructions to use the excel tool

In this section the calculation procedure to evaluate the capacity of the existing and reinforced masonry wall will be presented. The required information needed to run the calculation are resumed as follows:

- Geometry of the wall;
- Mechanical properties of the existing masonry;
- Forces acting on the wall;
- Geometry of the reinforcement;
- Mechanical properties of the reinforcement.

Step 1. Input geometrical parameters for the unreinforced wall

In this part of the calculation process the geometry of the wall according to Figure 1 has to be defined.

Input data

- Thickness of the masonry wall - t_m [mm]
- Width of the masonry wall - a [mm]
- Height of the masonry wall - h [mm]

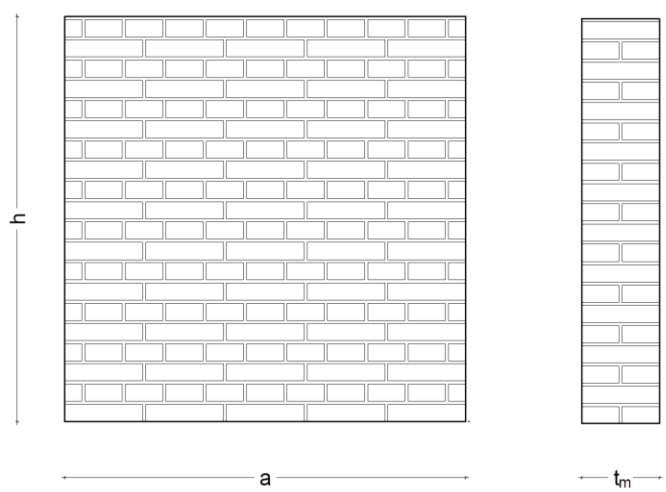

Figure 1 – Unreinforced wall

Step 2. Input mechanical parameters for the unreinforced wall

In this part of the calculation process the mechanical parameters of the wall have to be defined.
The reference values of the mechanical parameters and average specific weights suggested by the Italian standard NTC 2018 for selected types of masonry are reported in Table 1.
The proposed method is validated only for solid clay brick masonry. Therefore, the input mechanical parameters should stay in the range suggested in Table 1 (a warning will appear for values out of the ranges).

Masonry typology	f_m	τ_0	f_{v0}	E	G	w
	[N/mm^2]	[N/mm^2]	[N/mm^2]	[N/mm^2]	[N/mm^2]	[kN/m^3]
Disarranged masonry of cobbles/boulders	1,0 – 2,0	0,018-0,032	-	690-1050	230-350	19
Masonry of rough-hewed stones	2,0	0,035-0,051	-	1020-1440	340-480	20
Masonry of cut stones	2,6 - 3,8	0,056-0,074	-	1500-1980	500-660	21
Masonry of irregular soft stones	1,4 – 2,2	0,028-0,042	-	900-1260	300-420	13÷16
Masonry of squared soft stones	2,0 – 3,2	0,04-0,08	0,10-0,19	1200-1620	400-500	13÷16
Masonry of squared stone blocks	5,8 – 8,2	0,09-0,12	0,18-0,28	2400-3200	800-1100	22
Brickwork of solid blocks and lime mortar	2,6 – 4,3	0,05-0,13	0,13-0,27	1200-1800	400-600	18
Brickwork of semisolid blocks and cement mortar	5,0 - 8,0	0,08-0,17	0,20-0,36	3500-5600	875-1400	15

Table 1 – Mechanical properties for different masonry typologies as suggested by NTC 2018

Input data

- Masonry specific weight - γ_m [kN/m^3]
- Masonry compressive strength, mean value - f_m [MPa]
- Masonry shear strength, mean value (diagonal crack) - τ_0 [MPa]
- Masonry shear strength, mean value (stair-stepped crack) - f_{v0} [MPa]
- Knowledge level - KL [-]
- Masonry safety factor - γ_M [-]

Remark

The choice of the mechanical properties within the suggested range depends on the level of knowledge KL that has been reached. For KL = 1, the minimum value of the range should be selected. For KL = 2, the mean value of the range should be selected. For KL = 3, the value should be defined considering the results of the experimental tests performed according to § C8.5.4.1 of Italian Standard (Circolare NTC 2018) [1].

For the evaluation of the design values of the mechanical properties, the most appropriate knowledge level must be considered with the respective confidence factor CF as suggested by the Circolare NTC 2018 (Table 2). A safety factor CF is associated to each level of knowledge.

Knowledge level	Geometry	Details	Materials	Analysis	CF
KL1	From original architectural drawings with sample	Simulated design according to relevant practice and from **limited** *in-situ* inspection	Default values according to standards of the time of construction **and** from **limited** *in-situ* testing	Static linear analysis	1.35

KL2	visual survey or from full survey	From incomplete original executive construction drawings with **limited** in situ inspection <u>Or</u> From extended *in-situ* inspection	From original design specification with **limited** *in-situ* testing <u>Or</u> From **extended** *in*-situ testing	All	1.2
KL3		From original executive construction drawings with **limited** *in-situ* inspection <u>Or</u> From **comprehensive** *in-situ* inspection	From original test reports with **limited** *in-situ* testing <u>Or</u> From **comprehensive** *in situ* testing	All	1

Table 2 – Level of knowledge and respective confidence factor as per NTC 2018 [1]

If an elastic behavior with behavior factor q is assumed, the design strength values are achieved by reducing the mean values using both the confidence factor CF and the material safety factors γ_M. In case of non-linear calculation, only the confidence factor CF should be considered, by assuming the material safety factor γ_M equal to 1.

For the evaluation of material safety factor γ_M, it is possible to consider the values proposed by Tab. 4.5.II in § 4.5.6.1 of Italian Standard (NTC 2018) [2] (reported in Table 3). Moreover, according to section C8.7.1 of [1], in case of seismic verification, γ_M should be assumed equal to 2.

Material	**Class of execution control**	
	1	**2**
units of category I, mortar with guaranteed performance	2,0	2,5
units of category I, mortar with prescribed composition	2,2	2,7
units of category II, any mortar	2,5	3

Table 3 – Values of γ_M from Table 4.5.II of NTC 2018 [2]

Notes:

Class 2 of execution control should be assumed whenever:

- supervision and control of the workmanship is made by a qualified professional, employee of the contractor;
- supervision, control and inspection of the workmanship is made by a qualified professional, independent from the contractor.

Class 1 of execution control should be assumed whenever, in addition to the above controls:

- in-situ tests are carried out on the mortar and concrete;
- factory made mortar are used or, in case of if in-situ mixed mortar, appropriate measuring containers are used and the mixing is made under control.

Mechanical parameters of the wall: calculated design values

<u>Automatically calculated</u>

- Confidence factor - CF [-]
- Masonry compressive strength, design value - $f_{m,d}$ [MPa]
- Masonry shear strength, design value (diagonal crack) - $\tau_{0,d}$ [MPa]
- Masonry shear strength, design value (stair-stepped crack) - $f_{v0,d}$ [MPa]

Step 3. Input loads acting on the wall

In this part of the calculation process the forces acting on the wall are defined according to Figure 2. These values should be taken from an evaluation with an appropriate software or a manual calculation that considers all the loads acting on the structure.

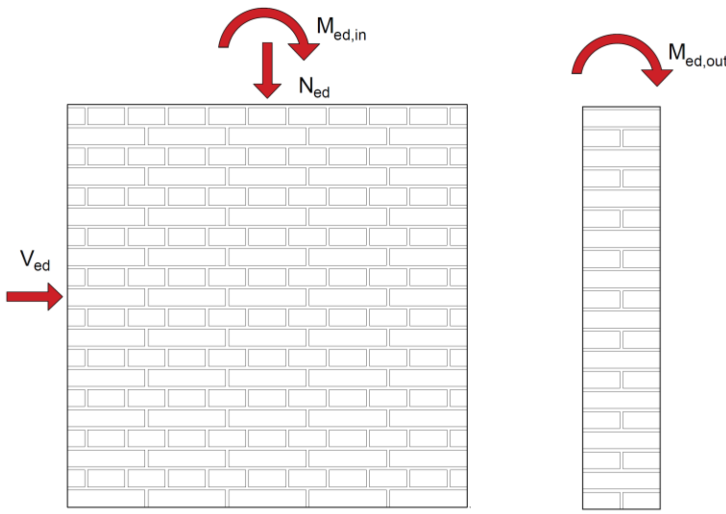

Figure 2 – Forces acting on the wall

Input data

- Axial compressive load – N_{ed} [kN]
- In-plane bending moment – $M_{ed,in}$ [kNm]
- Out-of-plane bending moment – $M_{ed,out}$ [kNm]
- Shear load – V_{ed} [kN]

Step 4. Input parameters of the reinforcement

In this part of the calculation process the properties of the reinforcement are defined, based on Figure 3 and Figure 4.

Input data

- Plaster thickness – t_p [mm] (the value must be in the range from 30 to 50 mm)
- Number of anchors per square meter – n [1/m²] (the value must range from 4 to 10)
- Anchor minimum edge distance – s_{min} [mm]
- Plaster specific weight – γ_p [kN/m³]
- Plaster compressive strength, mean value – f_c [MPa] (the value must range from 15 to 35 MPa)

Reinforcement parameters: assigned and automatically calculated values

Not modifiable data

- Anchor diameter – d [mm] (steel reinforcement bar of size 8 mm)
- Embedment depth of the anchor – h_{ef} [mm] (200 mm)
- Outside bend radius – R [mm] (24 mm)

Automatically calculated

- Extending/ protruding length – ΔL [mm]*
- Total straight length of the anchor (rebar; before bending) – L [mm]
- Anchors horizontal spacing – s [mm]
- Anchors vertical spacing – s_v [mm]
- Effective area of the wall to place anchors – A_e [m²]
- Number of anchor rows – r_n [-]
- Number of anchor columns – c_n [-]
- Total number of anchors on the wall (considering the edges) – n_{tot} [-]
- Number of anchors per square meter (considering the edges) – n_e [1/m²]
- Plaster elastic modulus, mean value – E_p [N/mm²]

*The parameter depends on the thickness of the plaster, and it results in negative values, if the plaster thickness is t_p < 40 mm. A negative value means that the bent part of the rebar extends into the masonry so that the total straight length is reduced keeping constant the embedment depth.

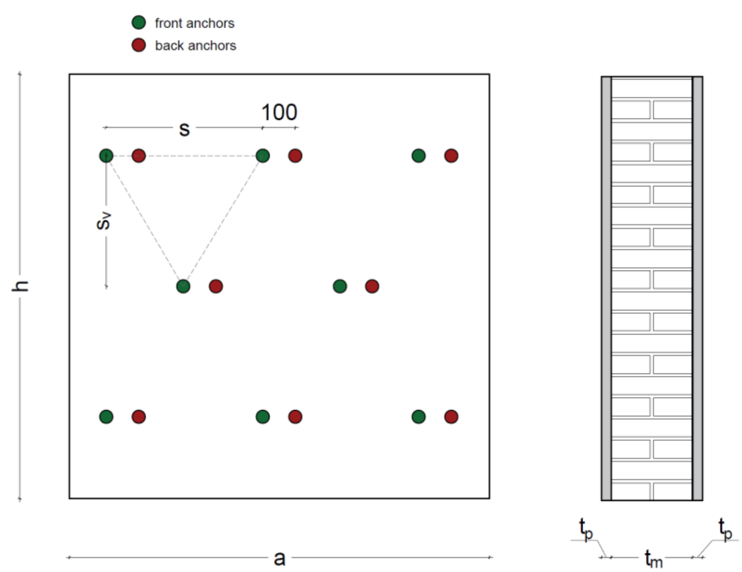

Figure 3 - Anchor configuration

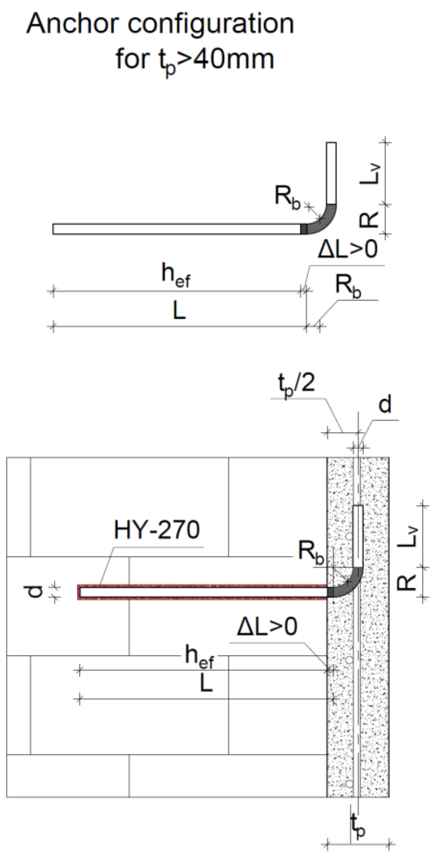

Figure 4 - Anchor detail

Output 1. Capacity of the unreinforced wall

All the values are automatically calculated from the input data according to the formulations proposed by the Italian Standard NTC [1,2].

- Maximum axial load - N_{rd}
- Maximum in-plane bending moment - $M_{rd,in}$
- Maximum out-of-plane bending moment - $M_{rd,out}$
- Maximum shear load, diagonal cracking - $V_{rd,1}$
- Maximum shear load, stair-stepped - $V_{rd,2}$

Output 2. Mechanical parameters of the retrofitted wall

All the values are automatically calculated from the input data. The mechanical properties are evaluated by applying the obtained increasing factor to the unreinforced masonry properties. For comparison, the properties obtained according to the Italian standard NTC [1] are also evaluated. The adopted NTC increasing coefficient is the maximum allowed by the Standard.

- Increasing factor – λ [-]
- Reinforced masonry compressive strength, design value - $f_{m,rd,retro}$ [MPa]
- Reinforced masonry shear strength, design value (diagonal crack) - $\tau_{0,rd,retro}$ [MPa]

- Reinforced masonry shear strength, design value (stair-stepped crack) - $f_{v0,rd,retro}$ [MPa]

Output 3. Capacity of the retrofitted wall

All the values are automatically calculated from the input data . The capacity of the strengthened wall is evaluated considering the increased mechanical properties. An automatic verification is made for each value, by comparing the capacity with the acting forces. When the verification is satisfied, the cell is highlighted in green, otherwise in red.

Automatic verifications

- Maximum axial load - $N_{rd,retro}$
- Maximum in-plane bending moment – $M_{rd,in,retro}$
- Maximum out-of-plane bending moment - $M_{rd,out,retro}$
- Maximum shear load, diagonal cracking - $V_{rd,1,retro}$
- Maximum shear load, stair-stepped – $V_{rd,2,retro}$
- Specific weight of retrofitted wall – $\gamma_{m,retro}$ [kN/m^3]

Bill of materials

- Number of anchors
- Total length of rebars ϕ12 mm
- HIT-HY 270 injection mortar
- Number of cartridges 500 ml
- Steel mesh, 10x10 cm, φ6 mm
- Plaster

The button **PRINT REPORT** can be used to save a pdf version of a report which contains all the main information about the input data, the calculation and the output data.

Application

Considering the masonry façade in Figure 5, the aim of the application is to evaluate the capacity of the pier highlighted in red, in the unreinforced and strengthened configuration.

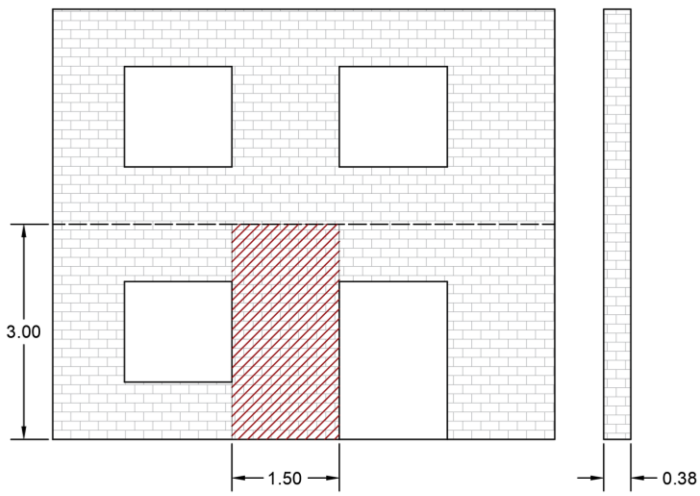

Figure 5 – Clay brick masonry wall (measures in meters)

Step 1. Input geometrical parameters for the unreinforced wall

Thickness of the masonry wall	t_m	380	[mm]
Width of the masonry wall	a	1500	[mm]
Height of the masonry wall	h	3000	[mm]

Step 2. Input mechanical parameters for the unreinforced wall

Masonry specific weight	γ_m	18.0	[kN/m³]
Masonry compressive strength, mean value	f_m	2.60	[N/mm²]
Masonry shear strength, mean value (diagonal crack)	τ_0	0.050	[N/mm²]
Masonry shear strength, mean value (stair-stepped crack)	f_{v0}	0.130	[N/mm²]
Knowledge level	KL	3	[-]
Masonry partial safety factor	γ_M	1.00	[-]

Mechanical parameters of the wall: calculated design values

Confidence factor	CF	1.00	[-]
Masonry compressive strength, design value	$f_{m,d}$	2.60	[N/mm²]
Masonry shear strength, design value (diagonal crack)	$\tau_{0,d}$	0.050	[N/mm²]
Masonry shear strength, design value (stair-stepped crack)	$f_{v0,d}$	0.130	[N/mm²]

Step 3. Input loads acting on the wall

Axial compressive load	N_{ed}	55.0	[kN]
In-plane bending moment	$M_{ed,in}$	34.0	[kNm]
Out-of-plane bending moment	$M_{ed,out}$	7.0	[kNm]
Shear load	V_{ed}	56.0	[kN]

Step 4. Input parameters of the reinforcement

Plaster thickness	t_p	30	[mm]
Number of anchors per square meter	n	5	[1/m²]
Anchor minimum edge distance	s_{min}	50	[mm]
Plaster specif weight	γ_p	24	[kN/m³]
Plaster compressive strength, mean value	f_c	35.00	[N/mm²]

Reinforcement parameters: assigned and automatically calculated values

Plaster thickness	t_p	30	[mm]
Number of anchors per square meter	n	5	[1/m²]
Plaster specif weight	γ_p	24	[kN/m³]
Plaster compressive strength, mean value	f_c	35.00	[N/mm²]
Anchor diameter	d	8	[mm]
Embedment depth	h_{ef}	200	[mm]
Extending/ protruding length	$\Delta L^{(1)}$	-5	[mm]
Outside bend radius	R	24	[mm]
Total straight length of the rebar	L	219	[mm]
Horizontal spacing	s	361	[mm]
Vertical spacing	s_v	400	[mm]
Anchor minimum edge distance	c_{min}	150	[mm]
Shift between the anchors on the two sides	c_{sides}	100	[mm]
Number of anchor rows	r_n	7	[-]
Number of anchor columns	c_n	4	[-]
Total number of anchors on the wall (considering the edges)	n_{tot}	14	[-]
Number of anchors per squaremeter (considering the edges)	n_e	4.67	[1/m²]
Plaster elastic modulus, mean value	E_p	25049	[N/mm²]

Output 1. Capacity of the unreinforced wall

Maximum axial load	N_{rd}	553	[kN]
Maximum in-plane bending moment	$M_{rd,in}$	24.8	[kNm]
Maximum out-of-plane bending moment	$M_{rd,out}$	6.19	[kNm]
Maximum shear resistance, diagonal cracking	$V_{rd,1}$	24.8	[kN]
Maximum shear resistance, stair-stepped	$V_{rd,2}$	29.7	[kN]

Output 2. Mechanical parameters of the retrofitted wall

		NTC	HILTI	
Increasing factor	λ	1.50*	1.93	[-]
Reinforced masonry compressive strength, design value	$f_{m,rd,retro}$	3.90	5.01	[N/mm²]
Reinforced masonry shear strength, design value (diagonal crack)	$\tau_{0,rd,retro}$	0.075	0.096	[N/mm²]
Reinforced masonry shear strength, design value (stair-stepped crack)	$f_{v0,rd,retro}$	0.195	0.251	[N/mm²]

Output 3. Capacity of the retrofitted wall

		NTC	HILTI	
Maximum axial load	$N_{rd,retro}$	1028	1321	[kN]
Maximum in-plane bending moment	$M_{rd,in,retro}$	26.0	26.4	[kNm]
Maximum out-of-plane bending moment	$M_{rd,out,retro}$	8.07	8.17	[kNm]
Maximum shear resistance, diagonal cracking	$V_{rd,1,retro}$	37.3	44.6	[kN]
Maximum shear resistance, stair-stepped	$V_{rd,2,retro}$	42.6	50.6	[kN]
Specific weight of retrofitted wall	$\gamma_{m,retro}$	19.16		[kN/m³]

Output 4. Increment of capacity/weight of the retrofitted wall

		NTC	HILTI	
Axial load	$N_{rd,retro}$	86%	139%	[kN]
In-plane bending moment	$M_{rd,in,retro}$	5%	6%	[kNm]
Out-of-plane bending moment	$M_{rd,out,retro}$	30%	32%	[kNm]
Shear resistance, diagonal cracking	$V_{rd,1,retro}$	51%	80%	[kN]
Shear resistance, stair-stepped	$V_{rd,2,retro}$	44%	70%	[kN]
Specific weight of retrofitted wall	$\gamma_{m,retro}$	6%		[kN/m³]

Theoretical background

Introduction

The proposed design approach is based on the results of an experimental program of diagonal compression tests performed on unreinforced and strengthened clay brick masonry walls [3] and aims to analytically evaluate the shear strength of walls reinforced with steel reinforced plaster (SRP) and subjected to diagonal compression test.

For the evaluation of the mechanical properties of the strengthened masonry, a standard square specimen with dimension of 1.3x1.3m (as suggested by ASTM 519 standard [4]) is considered and the limit load associated to a diagonal compression test is evaluated. The thickness of the specimen is defined according to the thickness of the masonry wall that must be verified. From the comparison between the maximum diagonal loads obtained for the unreinforced and reinforced specimens, an increasing factor is defined to evaluate the mechanical properties of the strengthened masonry. The latter will be used for all the reinforced masonry verifications.

Assumptions

During the diagonal compression test on the SRP masonry wall, two main phases can be differentiated:

a. the composite section (masonry + reinforced plaster) behaves like a monolithic section;
b. the reinforced concrete plaster delaminates, and the forces are redistributed according to the stiffness of the different materials; in this phase the anchors play a significant role to avoid instability phenomena.

To evaluate the shear strength of the wall, some simplifying assumptions were made:

1. The wall is modeled as an equivalent diagonal strut. This assumption was developed and adopted by several authors to describe, in a simple way, the behavior of infill masonry panels subjected to horizontal loads [5,6]. The experimental evidence proved that a frame, subjected to horizontal loads, tends to detach from the masonry infill in the vicinity of two opposite diagonal corners. The other two corners instead remain in contact with the frame, so that the masonry infill is subjected to diagonal compression and its structural behavior may be represented as a diagonal strut. The same concept has been extended to the masonry panels subject to diagonal compression test, in which the opposite corners along the compressed diagonal are constrained by the loading system, and the other two are free to deform. For the definition of the width w of the equivalent strut, the recommendation proposed by Stafford Smith [7] is followed, which suggests, based on some experimental data, that w should range between 0.15 and 0.25 of the strut length. In particular, the width of the strut was defined as:

$$w = 0.15L_d \tag{1}$$

with L_d length of the compressed diagonal (Figure 6).

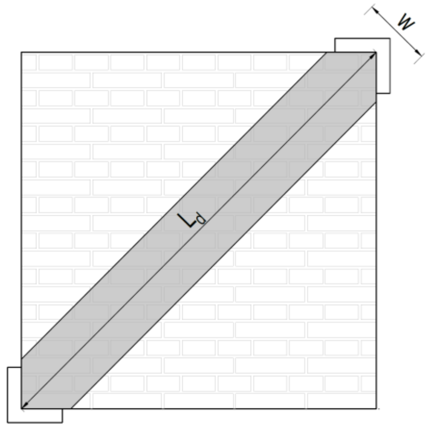

Figure 6 – Diagonal strut used to represent the wall in the analytical model.

2. An elastic-plastic behavior for the plaster and an elastic-brittle behavior for the masonry are considered as reported in Figure 7, where δ_m and δ_p are the generic displacements (i.e. shortening or elongation of the strut) of masonry and plaster, respectively, $\delta_{m,u}$ is the masonry ultimate displacement, $\delta_{p,y}$ is the displacement at plaster yielding and N_p and N_m are the maximum axial loads in the plaster and masonry, respectively.

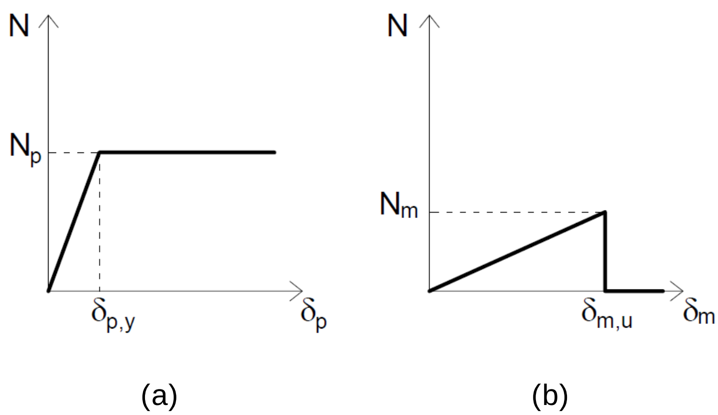

Figure 7 – Constitutive relations of (a) plaster, (b) masonry.

3. Before delamination occurs, plaster and masonry show compatible displacements:

$$\delta_m = \delta_p \tag{2}$$

Based on the latter assumption, masonry and plaster work "in parallel". Based on this and considering that the relation $\delta_{m,u} > \delta_{p,y}$ is satisfied for the most common masonry typologies, the capacity N_{max} of the composite section (Figure 8) may be evaluated as:

$$N_{max} = 2N_p + N_m \tag{3}$$

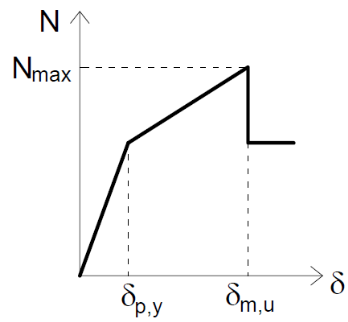

Figure 8 – Constitutive relation of the composite section.

Evaluation of the mechanical properties of the strengthened wall

In the following, the calculation of the contributions of the masonry, N_m, and the plaster, N_p, are given with the aim to evaluate the maximum load N_{max} associated to the diagonal compression test.

Masonry contribution

The masonry contribution N_m can be easily evaluated as:

$$N_m = f_m \cdot w \cdot t_m \tag{4}$$

where f_m is the masonry compressive strength, w is the width of the strut and t_m is the masonry thickness.

Plaster contribution

In the evaluation of the plaster contribution under compression, two different failure modes must be considered: 1) the failure for crushing; 2) the failure for instability, which may happen after the detachment of the plaster due to the high slenderness of the plaster layer.

1. Crushing failure

The limit load $N_{p,c}$ associated with the crushing of the plaster is calculated as:

$$N_{p,c} = f_{c,r} \cdot w \cdot t_p \tag{5}$$

where t_p is the plaster layer thickness and $f_{c,r}$ is its compressive strength, evaluated as:

$$f_{c,r} = v \cdot f_c \tag{6}$$

where v is a strength reduction factor, defined according to §6.2.2 of [8], which takes into account the effect of shear cracks on the compressive resistance f_c, and it is equal to:

$$v = 0.6\left(1 - \frac{f_c}{250}\right) \tag{7}$$

2. Instability failure

In the evaluation of the critical load associated to the instability phenomenon, the following elements are taken into account:

- Correction of the elastic properties due to second order effect;
- The presence of the anchors, which helps the plaster to prevent instability;
- The presence of initial imperfections, which reduce the theoretical instability load.

2.1 Correction of the elastic properties

The plaster elastic modulus is evaluated according to the ACI318-95 [9] as a function of the plaster compressive strength f_c:

$$E_p = 4234\sqrt{f_p} \tag{8}$$

According to NTC 2018 (§4.1.2.3.9.3) [1,2] and EC2 (§5.8.7.2) [8], in the evaluation of the instability load for concrete columns (plaster strut in the present case), the elastic modulus E must be corrected to take into account second order effects as follows:

$$E_{p,r} = \frac{0.3}{1+0.5\phi} E_p \tag{9}$$

Where E_p is the design plaster elastic modulus and ϕ is the viscosity coefficient which in the common practice is assumed equal to 2 (for the definition of see §11.2.10.7 of [2]).
[1,2][8]

2.2 Anchors effect

The presence of anchors between masonry and plaster reduces the effective length of the diagonal strut, so that in the evaluation of the critical load due to instability the contribution of the anchors is taken into account.
The anchors are considered as unilateral elastic supports, whose stiffness α is evaluated with an appropriate analytical formulation suggested by Papia and Russo [10,11] . Instead of considering discrete springs, the stiffness of the anchors is distributed along the whole strut length) as suggested in [11], in analogy with the Winkler soil method (Figure 9). The distributed approach has the advantage of avoiding the use of a numerical procedure to find the solution, as instead necessary using discrete springs.
The distributed stiffness β is defined as:

$$\beta = \frac{\alpha}{\ell} \tag{10}$$

where α is the anchor stiffness and ℓ is the maximum spacing between the anchors (assumed constant).

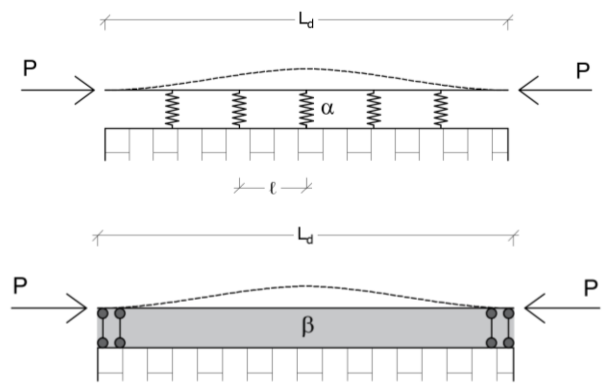

Figure 9 – Top, model with discrete springs. Bottom, model with distributed stiffness.

If experimental data are not available, the stiffness of each anchor α can be evaluated, considering both the axial and the bending deformability, by applying the force method to the structure of figure 1, where q are the bearing reaction forces. From the equilibrium rotation of the structure with F = 1, the bearing reaction forces can be expressed as q = (R-d/2)/Lq^2. The anchor stiffness is expressed in equation (11), in which h_{ef} is the embedment depth, R is the center line bending radius, E_s is the steel elastic modulus, A_s is the cross-section area of the rebar, and I_s is the second moment of area of the anchor section

(Figure 10). The length L_q on which the bearing pressure q acts is estimated as the minimum between 10d and $h_{ef}/2$, the length ΔL depends on the thickness of the plaster and it may assume negative values when $t_p < 40$ mm. A negative value means that the bent part of the rebar extends into the masonry so that the total straight length is reduced keeping constant the anchor embedment depth (which starts at the masonry surface by definition).

$$\alpha = \frac{1}{\frac{h_{ef}+\Delta L}{E_s A_s} + \frac{\pi\left(R-\frac{d}{2}\right)}{4E_s A_s} + \frac{\left(R-\frac{d}{2}\right)^2 L_q}{10E_s I_s} + \frac{\left(R-\frac{d}{2}\right)^2 L_q}{E_s I_s} - \frac{\left(R-\frac{d}{2}\right)^2 L_q}{3E_s I_s} + \frac{\left(R-\frac{d}{2}\right)^2 \Delta L}{E_s I_s} + \frac{\pi\left(R-\frac{d}{2}\right)^3}{4E_s I_s}} \tag{11}$$

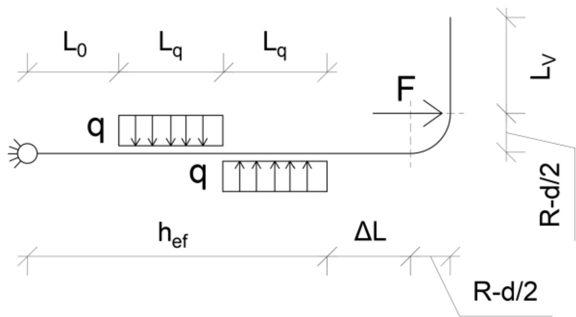

Figure 10 – Structure for the evalution of the anchor's stiffness.

The dimensionless critical load c_{cr} can be defined as a function of a parameter γ as:

$$c_{cr} = \frac{2}{\pi^2}\sqrt{3\gamma} \quad \text{(for distributed stiffness)} \tag{12}$$

with

$$\gamma = \frac{\alpha \ell^3}{E_{p,r} I_p} \tag{13}$$

Figure 11 shows the relation between c_{cr} and γ.

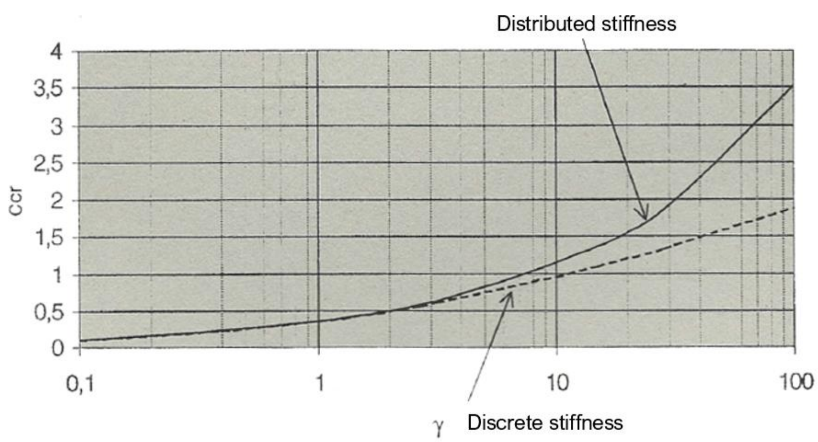

Figure 11 – Dimensionless critical load c_{cr} vs γ.

The critical load due to instability P_{cr}, which accounts for the presence of anchors, is obtained as:

$$P_{cr} = c_{cr} P_e \tag{14}$$

where P_e is the Euler's critical load, defined as:

$$P_e = \frac{\pi^2 E_{p,r} I_p}{\ell^2} \tag{15}$$

2.3 Imperfections

The presence of initial imperfections (e.g., imperfections from wall construction, eccentricity of the load) can further reduce the ultimate load. The imperfection can be represented as an initial deflection shape $\bar{v}(x)$ of the diagonal strut, which can be expressed as in equation (16), where L_d is the strut length, U_ξ is the maximum initial displacement at the midpoint and P_{cr} is the critical load which accounts for the presence of anchors, evaluated via equation (14). The final deflection curve $v(x)$ is influenced by the initial curvature and is reported in equation (17), where $N_{p,i}$ is the maximum load considering both the anchor's effect and the initial imperfection.

$$\bar{v}(x) = U_\xi \sin\frac{\pi x}{L_d} \tag{16}$$

$$v(x) = U_\xi \frac{1}{1 - N_{p,i}/P_{cr}} \sin\frac{\pi x}{L_d} \tag{17}$$

The final maximum displacement U_{max} can be evaluated as:

$$v\left(\frac{L_d}{2}\right) = U_{max} = U_\xi \frac{1}{1 - N_{p,i}/P_{cr}} \tag{18}$$

The critical load which account for both imperfections and instability is evaluated from eq. (18) as:

$$N_{p,i} = i \cdot P_{cr} \tag{19}$$

where the reduction coefficient i is defined as:

$$i = \left(1 - \frac{U_\xi}{U_{max}}\right) \tag{20}$$

The initial imperfection U_ξ can be evaluated according to Eurocode 2 (§4.3.5.4) [8] stating that "for individual structural elements (not frame), geometrical imperfections may be taken into account by increasing the eccentricity of the load with an additional eccentricity". The additional eccentricity e_a is defined as:

$$U_\xi = e_a = \frac{\nu \ell}{2} \tag{21}$$

where ℓ is the effective length (spacing between anchors expressed in meters) and ν is defined as:

$$\nu = \frac{1}{100\sqrt{\ell}} \quad \text{(rad)} \tag{22}$$

with n_{min}=1/200 as lowest value to be inserted according to Eurocode 2 (§4.3.5.4) [8].
The maximum deflection U_{max} is evaluated according to the following analytical expression:

$$U_{max} = (\varepsilon \cdot t_p + m) n^{\left(\delta \frac{t_p}{t_{ref}} - \omega\right)} \tag{23}$$

where t_p is the plaster thickness, n is the number of anchors per square meter and ε, μ, δ, ω and t_{ref} are constants, calibrated on the bases of numerical simulations, with the following values:

ε = -0.0067
μ = 16.377 mm
δ = 0.13
ω = 0.2773
t_{ref} = 100 mm

The instability critical load of the plaster, $N_{p,i}$, accounting for both, anchors and imperfections, is finally evaluated as:

$$N_{p,i} = P_e \cdot c_{cr} \cdot i = \sqrt{\frac{12\alpha EI}{\ell}}\left(1 - \frac{U_\xi}{U_{max}}\right) \tag{24}$$

Limit load for reinforced masonry

The resulting ultimate load for the reinforced masonry N_{max} is:

$$\begin{aligned} N_{max} &= 2 \cdot N_{p,i} + N_m \quad \text{if} \quad N_{p,c} > N_{p,i} \\ N_{max} &= 2 \cdot N_{p,c} + N_m \quad \text{if} \quad N_{p,c} \le N_{p,i} \end{aligned} \tag{25}$$

Mechanical properties of the strengthened wall

According to the approach suggested by NTC 2018 [2], the mechanical properties of the reinforced wall are evaluated applying an increasing factor to the initial values of the unreinforced masonry.
The increasing factor λ is evaluated from the comparison between the limit load N_m associated to the unreinforced masonry and load N_{max} which accounts for both, masonry and plaster:

$$\lambda = \frac{N_{max}}{N_m} \tag{26}$$

The mechanical properties of the reinforced masonry are calculated as follows:

$$f_{m,r} = f_m \cdot \lambda \tag{27}$$

$$\tau_{0,r} = \tau_0 \cdot \lambda \tag{28}$$

$$f_{v0,r} = f_{v0} \cdot \lambda \tag{29}$$

Masonry wall verifications

The verifications on the masonry wall are performed according to the NTC 2018 provisions [2], using the mechanical properties defined in equations (27)-(29).
The maximum axial load without bending moment for the masonry cross-section is:

$$N_{rd} = 0.85 \cdot a \cdot t \cdot f_{m,d} \tag{30}$$

where a is the length of the wall cross-section, t is the thickness of the compressed masonry and $f_{m,d}$ is the design compressive strength.
The maximum bending moment for the masonry cross-section is:

$$M_{rd} = \frac{a^2 \cdot t \cdot \sigma_0}{2}\left(1 - \frac{\sigma_0}{0.85 f_{m,d}}\right) \tag{31}$$

where a is the length of the wall cross-section, t is the masonry thickness, σ_0 is the average vertical axial load and $f_{m,d}$ is the design compressive strength.
In case of masonry with a regular bond, the maximum shear load can be evaluated considering two different failure criteria: the stair-stepped diagonal cracking and the isotropic diagonal cracking (Figure 12). This last one is more conservative and does not take into account the masonry bond.

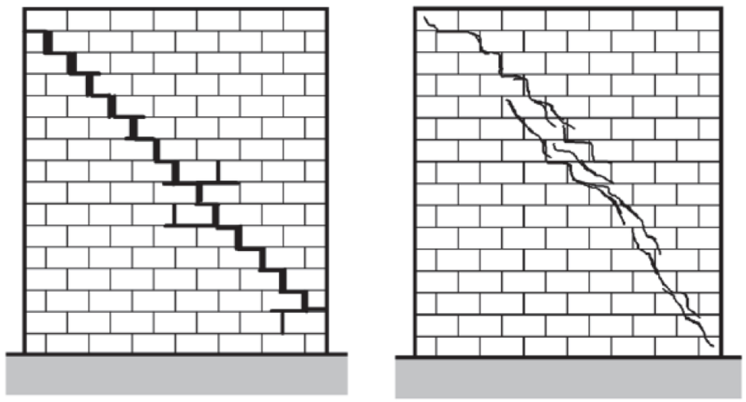

Figure 12 – Left, stair-stepped cracking. Right, isotropic diagonal cracking.

Considering the diagonal cracking criteria, the maximum shear load is:

$$V_{rd,1} = a \cdot t \frac{1.5 \cdot \tau_{0,d}}{b} \sqrt{1 + \frac{\sigma_0}{1.5 \cdot \tau_{0,d}}} \tag{32}$$

where a is the length of the wall cross-section, t is the masonry thickness, b is the stress distribution factor (evaluated as H/h with H vertical dimension of the wall and h height of the cross-section, with value between 1 and 1.5, see §C8.7.1.3.1.1 in [1]), σ_0 is the average vertical axial load, $\tau_{0,d}$ is the design shear strength for diagonal cracking.

Considering the stair-stepped failure criteria, the maximum shear load is:

$$V_{rd,2} = \frac{a \cdot t}{b} \left(\frac{f_{v0d}}{1 + \mu\phi} + \frac{\mu}{1 + \mu\phi} \sigma_0 \right) \tag{33}$$

where a is the length of the wall cross-section, t is the masonry thickness, s_0 is the average vertical axial stress, μ is the friction coefficient, ϕ is a bond coefficient and $f_{v0,d}$ is the design shear strength for stair-stepped cracking. Suggested values for μ and ϕ can be found in §C8.7.1.3.1.1 of [1].

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